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Exploring measurement estimation strategies through concept cartoons designed with Realistic Mathematics Education

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Concept cartoons, in relationship with Realistic Mathematics Education (RME), not only serve as a dynamic platform for problem-solving but also intricately weave mathematical concepts into the fabric of real-world scenarios, creating a harmonious fusion of theory and practical application. By using relatable characters and presenting ideas through engaging narratives, students are encouraged to tackle problems within the given context. The aim of this study was to investigate the measurement estimation strategies used by students when exposed to concept cartoons specifically designed for RME. A qualitative research methodology was followed, involving 46 fourth-grade students from a primary school. The data collection instrument utilized was "RME-Supported Concept Cartoons." Descriptive analysis was employed to analyze the collected data. The concept cartoon activities incorporated measurement estimation strategies such as prior knowledge, unit iteration/separation, segmentation/chunking, and comparison, which students could potentially prefer. However, it was observed that students predominantly utilized division into unit iteration/separation and segmentation/chunking strategies over other estimation strategies. Furthermore, the frequency of strategy use did not exhibit significant variations based on gender. When examining the strategies developed by the students, it is noteworthy that the presence of "another solution-oriented option" and "irrelevant answers" was prominent.

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Introduction

Representations play a vital role in enhancing students' understanding of mathematics by transforming complex problems into graphical, visual, symbolic, or alternative linguistic formats (National Council of Teachers of Mathematics [NCTM], 2000). Accomplished mathematicians have emphasized the effectiveness of visual representations in reasoning (Jitendra and Woodward, 2019), highlighting how they facilitate the depiction of imagination and employ diagrams and models to embody abstract information. In the field of education, visual representations are widely utilized by teachers, particularly in primary schools, as a strategy to capture students' attention during the learning process (NCTM, 2000; Aldous, 2007).

Concept cartoons serve as one method through which teachers employ visual representations. By utilizing concept cartoons, teachers not only make abstract mathematical concepts more concrete but also engage students through the use of visually appealing illustrations and educational scenarios. Concept cartoons, in the form of illustrated arguments, facilitate active participation from all students by introducing different perspectives within familiar environments through speech bubbles (Dabell, 2008). These cartoons not only promote new and original learning but also foster discussion and critical thinking among students.

Cartoons play a significant role in mathematics by bridging the gap between mathematical concepts and real-world applications (Clair, 2018). The use of realistic characters in cartoons discussing and reviewing mathematical ideas proves valuable in capturing students' attention. Moreover, concept cartoons align with the core principle of Realistic Mathematics Education (RME), which views mathematics as a human activity (Freudenthal, 1991, p. 14). The introduction of RME underscores its commitment not only to connecting mathematical concepts with real-world applications but also to fostering an environment where mathematical understanding becomes genuinely real for students (Çilingir Altiner, 2021). This includes providing meaningful contexts and problems that resonate with students' lived experiences, an aspect integral to RME (Fredriksen, 2021). The term 'realistic' in RME extends beyond its connection to the real-world; it involves creating contexts that are genuinely meaningful and relatable for students. Moreover, RME encompasses both horizontal and vertical mathematization (Treffers, 1993). Horizontal mathematization involves engaging with mathematical concepts in contexts that make sense to students, while vertical mathematization facilitates the transition from informal to more formal mathematical reasoning through dialogs and discussions. As such, RME represents an approach in which the aim is to make mathematics both accessible and comprehensible to learners (Bray and Tangney, 2015). Consequently, concept cartoons offer a beneficial strategy for RME, as they present problems derived from real-life situations that students need to solve. By utilizing concept cartoons, the transition from horizontal mathematization to vertical mathematization can be facilitated through dialogs and discussions.

Concept cartoons, as employed in this research, function as a method within the broader framework of RME (Gravemeijer, 1994). They are pedagogical tool strategically crafted to introduce problems stemming from real-life scenarios and promote active problem-solving. Featuring relatable characters and situations, concept cartoons aid in concretizing abstract mathematical concepts (Dabell, 2008), thus aligning with the fundamental principles of RME (reality principle of RME). By offering a visual and narrative-based approach, they enhance students' interaction with mathematical problems in the context of their daily experiences (the activity and interaction principle of RME), embodying both a method and strategy that facilitate the implementation of RME principles in mathematics education. Despite the diverse applications of concept cartoons in mathematics education, the literature lacks a comprehensive exploration of their alignment with the core tenets of RME (Van den Heuvel-Panhuizen, 2003; Freudenthal, 1991). In the present study, the aim is to address this gap by examining the potential of concept cartoons specifically tailored to adhere to RME principles.

RME is an educational approach that individuals frequently utilize in their daily lives, often without even realizing it. RME provides contexts meaningful and relevant to students' lived experiences. For instance, when we go grocery shopping, we engage in practical mental calculations and apply mathematical reasoning to make decisions, such as selecting a watermelon that will fit in a specific space or determining how many apples we can carry in a bag. These estimation calculations yield results that are close to actual measurements. Particularly, adults frequently rely on measurement estimation to generate quick solutions in their day-to-day activities. In a study focused on enhancing measurement estimation skills, it was found that instruction based on RME was more effective compared to traditional teaching methods (Kaba and Şengül, 2017). This highlights the practical relevance and effectiveness of incorporating RME principles in teaching measurement estimation skills to students.

In the present study, the measurement learning domain is addressed, which is known to be challenging for students and often involves misconceptions and difficulties related to conservation. Within this context, estimation is highlighted as a crucial aspect of learning measurement. Over the years, measurement estimation has become integral to the mathematics curriculum (Andrews et al., 2022). Understanding how to estimate measurements is of significant importance for two primary reasons.

Firstly, estimation is considered a fundamental arithmetic skill in its own right and is frequently employed in everyday life when precise measuring tools are unavailable or inappropriate (Levin, 1981; O'Daffer, 1979). Secondly, measurement estimation offers a practical approach to teaching physical measurement, a key topic within the elementary mathematics curriculum that lays the foundation for other mathematical concepts like fractions and ratios (Coburn and Shulte, 1986; Davydov and Tsvetkovich, 1991; NCTM, 2000). The NCTM has advocated measurement estimation as the central focus in preschool and elementary school mathematics curricula (NCTM, 1989, 2000). There are three primary reasons supporting the inclusion of measurement estimation in the curriculum: (1) Estimation practice helps students develop a mental reference for the dimensions of measurement units. (2) Estimation is a practical and valuable skill. (3) Estimation activities allow the application of fundamental measurement concepts.

However, it has been noted that limited knowledge exists regarding children's abilities in the area of measurement estimation, particularly at the primary school level (Ruwisch et al., 2015). Most research on measurement estimation has focused on upper-grade students, which prompted us in the present study to concentrate on lower-grade children (Sowder, 1992). The ability to estimate the length or area of an object with a reasonable degree of accuracy is a complex skill that requires a range of abilities and concepts, namely:

- understanding the feature (length or area) to be measured,
- understanding of the unit concept,
- a mental image of the unit used in the estimation task,
- ability to compare objects on the attribute to be measured,
- ability to perform unit iterations,
- ability to select and use appropriate strategies for estimation, and

- ability to check the relevance of the estimate.

Considering the complexity of skills primary school students need to develop and their limited ability to fully articulate the measurement estimation strategies they employ, the use of supported concept cartoons is predicted to be beneficial in identifying these strategies. However, when examining the existing literature, it is evident that concept cartoons are rarely used in mathematics education compared to in other fields (Aygün et al., 2020; Huang and Tzu-Ying, 2020; Sancar and Koparan, 2019). Most studies on concept cartoons in mathematics have focused on teacher-oriented concept cartoons and the effectiveness of concept cartoon-supported instruction in geometry lessons for secondary school students (Samková, 2020; Kogler et al., 2021; Karaca et al., 2020; Aygün et al., 2020; Sancar and Koparan, 2019; Göksu and Köksal, 2016; Şengül and Aydın, 2013). However, the relationship between concept cartoons and RME has not been explicitly explored in the literature.

Therefore, the aim of the present study is to determine the measurement estimation strategies utilized by fourth-grade primary school students through concept cartoons specifically designed with realistic problems. Concept cartoons can be applied in various ways and serve different purposes. Given the identified advantages of this approach, we seek to reveal students' existing knowledge enjoyably and ascertain the strategies they employ. The primary objective is to identify the measurement estimation strategies used by the students. Specifically, the following research questions will be addressed:

- What measurement estimation strategies do fourth-grade students use in concept cartoons related to weight, length, and area problems?
- Are there any differences in the measurement estimation strategies used by students in concept cartoons based on their gender?
- What measurement estimation strategies do students generate for weight, length, and area problems?

The intention is to uncover the strategies they independently produce to estimate measurements. By addressing these research questions, the aim is to provide insights into the measurement estimation strategies used by fourth-grade students in concept cartoons, examine potential gender differences, and explore their ability to generate strategies for weight, length, and area problems.

Methods

The research method chosen for this study is the case study, which is a qualitative research approach. This decision was made based on several factors. Firstly, the study involves examining an existing process, namely the measurement estimation strategies of fourth-grade primary school students in concept cartoons. Secondly, real-life problems were selected, allowing an in-depth exploration of their experiences and perspectives. Thirdly, the study involves a complex and multifaceted phenomenon with potentially blurred boundaries, as it investigates measurement estimation strategies in various contexts (weight, length, and area problems) within the concept cartoons.

Participants. The study included a total of 46 fourth-grade students (23 girls and 23 boys) from a school in Adana, Turkey, during the 2021–2022 academic year. The school comprised seven classrooms for fourth graders, with a total student population of 175. To ensure the reliability of the research, purposeful random sampling was employed (Flick, 2014). The selection of two classes was based on the willingness of the classroom teachers to participate in the study. Fourth-grade students were chosen because they have more experience in measurement estimation compared to students in other grade levels. Additionally, students in this age group possess the ability to think more abstractly and have more developed mental estimation skills. It was believed that fourth-grade students would be capable of developing their own estimation strategies. The age range of the participating students varied between 9 and 10 years, with an average age of 9 years and 6 months. The district where the school is located has a moderate economic level, resulting in a student population with similar socioeconomic backgrounds. The socioeconomic status of the school corresponds to that of middle-class families, as reported by the Turkish Statistical Institution (TUIK, 2021).

The study did not include students with learning difficulties requiring special education, students with exceptional academic achievements, or students facing difficulties in accessing the learning environment. Furthermore, there were no students from different ethnic groups in the selected classes, ensuring that the language development and literacy levels of the participating students were similar. The fact that the students were in the fourth-grade indicated a good level of literacy compared to other primary school grade levels.

Data collection tool. The data collection tool used in the study was RME-Supported Concept Cartoons developed by the researcher. Unlike generic Concept Cartoons, these specialized instruments are purposefully crafted to integrate seamlessly with the RME framework, aiming to create a more profound connection between mathematical concepts and real-world applications. The differentiation lies in the meticulous alignment of content and scenarios with the core tenets of RME, ensuring that the problems presented are not only relatable but also resonate authentically with students' lived experiences. Furthermore, the RME-Supported Concept Cartoons prioritize specific measurement-related scenarios, addressing essential learning outcomes in the primary school mathematics program, such as weight, length, and area measurement estimations. The scenarios embedded in these cartoons are deliberately designed to motivate students towards active problem-solving within the context of their daily lives. In essence, the tool serves as a targeted and tailored application of Concept Cartoons, specifically adapted to adhere to the principles of RME (for example, horizontal and vertical mathematization). This distinction ensures that the RME-Supported Concept Cartoons are not merely a generic pedagogical tool but a finely tuned and purpose-built instrument, strategically developed to enhance the integration of RME principles in the teaching and learning of mathematics. The alignment with RME goes beyond superficial visual elements, delving into the very fabric of scenario construction and problem presentation to create a more meaningful and effective learning experience for students.

After each scenario, the students were asked to identify which student in the cartoon they believed chose the most appropriate strategy for solving the problem. They were instructed to note their choice on the activity sheet provided. Furthermore, the students were asked to generate their own strategies for the given problem situation and record them. This allowed the researchers to gather insights into the measurement estimation strategies developed by the students themselves. Figure 1 provides an example of an RME-supported concept cartoon activity, illustrating the use of a realistic problem related to length measurement in the learning field.

Figure 1 illustrates a classroom environment with teachers and students. In the RME-supported concept cartoon activity, Elçin's answer indicates that she relied on her own experiences, prior knowledge, and previous measurements to answer the question.

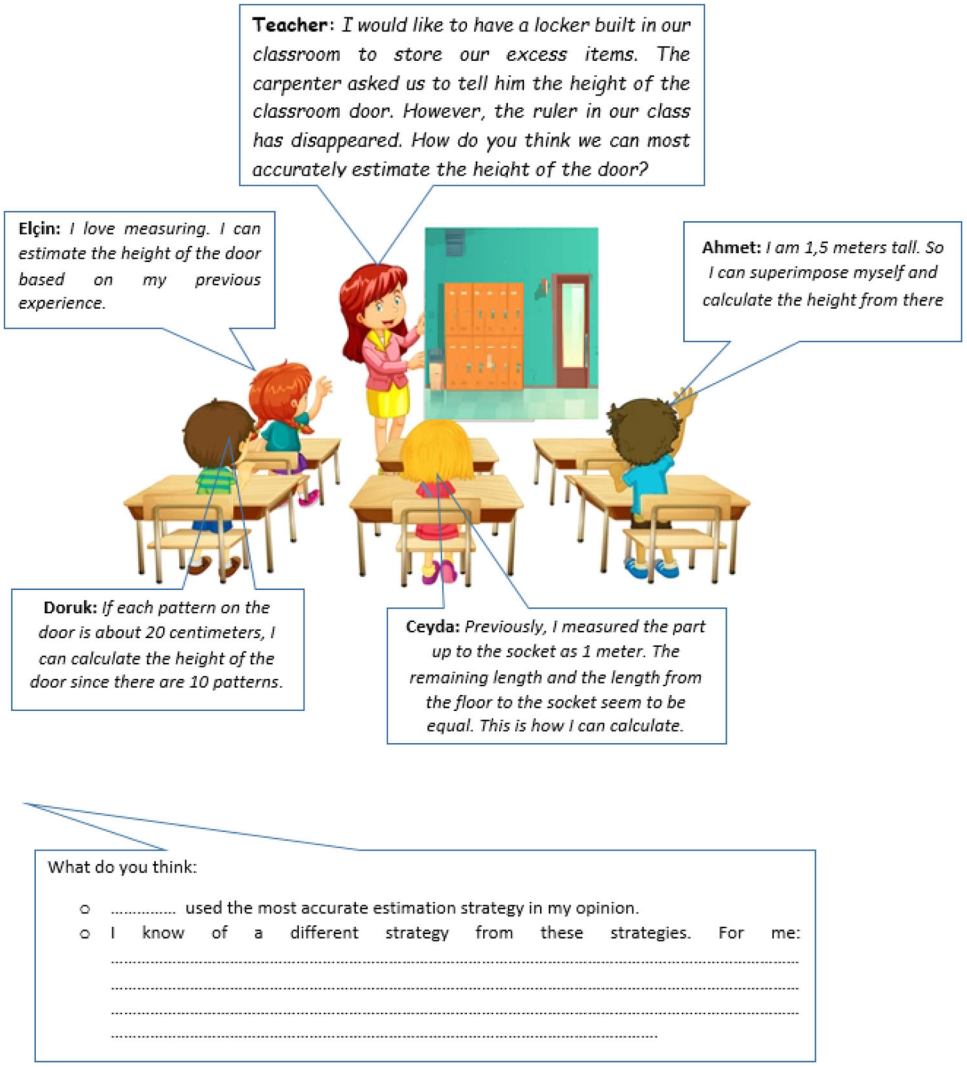

Fig. 1 Concept cartoon activity for the length measurement estimation problem.

This suggests that Elçin employed a strategy based on personal experiences and existing knowledge to estimate the measurement in the given problem situation.

In the RME-supported concept cartoon activity, Doruk employed the unitization strategy to estimate the size of the door. He calculated each pattern on the door as a unit and used this information to estimate the overall size. The unitization strategy involves mentally creating nonstandard units (e.g., span) and comparing them repeatedly to the object being estimated. Doruk needed to remember the image of the unit, track where the last unit ended, and determine where the next unit should begin (Joram et al., 2005).

On the other hand, Ceyda used the segmentation/subdividing/chunking strategy. She identified the part of the door up to the socket as a midpoint and stated that this part represented half the length of the door. In segmentation/chunking, smaller units are compared to a known length by dividing them into approximately two lengths instead of iterating them one by one. This strategy allows a reduction in the number of unit iterations required for estimation.

Ahmet employed the comparison strategy by assessing the height of the door in relation to his own height. Through this approach, he inferred that the door was taller than him and made an estimation accordingly. The comparison strategy, also known as the reference point strategy, involves mentally envisioning an object with a known measurement, such as a 3-cm-long eraser, and comparing it to the object being estimated, such as the length of a book. This strategy relies on individuals having a mental reference unit or a conceptual understanding of the size of the unit (Sowder, 1992, as cited in Joram et al., 2005). For instance, a dermatologist might utilize a reference point of a "pencil eraser" when advising patients to be cautious of moles larger than 6 mm (approximately the size of a pencil eraser). This reference point is chosen because it is a familiar object. By establishing a connection between a numerical value and a known quantity, reference points enhance the meaningfulness of units of measurement for estimation purposes.

Data collection process. A 4-week study was conducted in two selected classes after approval was obtained the school administration and permission from the classroom teachers. The study adhered to the existing fourth-grade mathematics curriculum in primary schools. The weeks allocated for teaching the measurement learning area were chosen for implementing the study. During the first week of the intervention, the concept cartoons were introduced to the students by their classroom teacher. The teacher provided information about the purposes and uses of concept cartoons. Sample concept cartoon exercises were distributed and preparations were made for the actual application of the study.

Table 1 Measurement estimation strategies used by students.

Estimation strategies	Weight measurement estimation		Length measurement estimation		Area measurement estimation		Total	
	f	%	*f*	%	*f*	%	*f*	%
Reference point/comparison	12	26.1	9	19.6	8	17.4	29	21
Segmentation/subdividing/chunking	26	56.5	11	23.9	16	34.8	53	38.4
Prior knowledge	5	10.9	6	13.0	13	28.3	24	17.3
Unit iteration/separation	3	6.5	20	43.5	9	19.6	32	23.1

The main application took place once a week during regular class hours. The students were given concept cartoon worksheets that contained realistic problems related to weight, length, and area in the measurement learning field. The teacher read the problem presented on the worksheet to the students, who were then asked to indicate their chosen answer on the worksheet. Additionally, the students were invited to share any alternative strategies they might have by noting them on the worksheet. This process was repeated for all other problem scenarios. Throughout the application, the classroom teacher acted as an observer and was present in the classroom. It is important to note that the study strictly adhered to ethical guidelines. In line with ethical guidelines, the study was designed to ensure the protection and well-being of the participants. Necessary approvals, such as parental consent forms and school board information forms, were obtained before conducting the research. The students were explicitly informed about the study's purpose and their participation was voluntary. They were assured that their responses would be treated as confidential and solely used for research purposes. Furthermore, the students were informed that they had the right to withdraw from the study at any point if they wished to do so. These measures were put in place to uphold ethical standards and ensure the rights and privacy of the participants.

Analysis of data. The data collected were subjected to descriptive analysis, following the guidelines outlined by Yıldırım and Şimşek (2021). Descriptive analysis aims to present individuals' opinions and experiences directly, without any added comments, by analyzing data gathered from interview transcripts, document texts, and observation notes. Direct quotations are often used to support the findings. In the present study, the strategies utilized by the students, as indicated by the names they selected from the concept cartoon worksheets, were coded. The frequency of these strategies' use and potential differences based on gender were examined. Additionally, the students were asked if they had any strategies of their own for solving the concept cartoons, and these self-generated strategies were analyzed. All strategies were coded systematically. Throughout the analysis process, the researcher maintained objectivity and refrained from expressing personal opinions or comments. The data were presented to the reader without any additional bias or interpretation.

To enhance reliability in qualitative research, member checking is often considered a valuable method (Miles and Huberman, 2016). In the present study, an additional step was taken to ensure reliability by involving another researcher who is an expert in mathematics education to analyze the data and obtain the results. This approach allows independent analysis and helps mitigate potential bias or subjective interpretations. To assess the consistency between the two analyses, Miles and Huberman's (2016) reliability formula was employed: Reliability = Consensus/(Agreement + Disagreement) × 100. The calculation resulted in a consistency value of 98%. This high consistency value indicates a strong agreement between the two researchers' analyses, further reinforcing the reliability of the research findings. According to Miles and Huberman (2016), reliability calculations exceeding 70% are generally considered reliable for qualitative research. In this case, with a reliability value of 98%, the research findings exhibit a high level of reliability based on this criterion.

Results

The first analysis of the research is about which of the existing estimation strategies students use. In Table 1, the strategies chosen most by the students in the face of real-life problems through concept cartoons are given.

Upon examining Table 1, it is evident that the students displayed varying preferences for estimation strategies when solving different measurement problems. In the weight measurement problem, the segmentation/chunking strategy was the one used most among the estimation strategies produced ($F = 26$). Following that, the comparison strategy was the second most frequently used strategy ($F = 12$). However, the pattern of students' choices changed when it came to the length measurement problem. In this case, the unit iteration/separation strategy was predominantly favored by the students ($F = 20$). This indicates a shift in their approach to estimation when dealing with length measurements. For the area measurement problem, similar to the weight measurement problem, the segmentation/subdividing/chunking strategy emerged as the most popular strategy ($F = 16$). Additionally, the experience strategy was also notable among those chosen ($F = 13$).

These findings highlight the varying strategies employed by the students when estimating measurements in different contexts. The segmentation/chunking strategy appears to be consistently favored in weight and area measurements, while the unit iteration/separation strategy takes precedence in length measurements. The presence of the experience strategy in the area measurement problem suggests that students may draw upon their prior knowledge and familiarity with certain contexts when estimating areas. The measurement strategies used by the students according to their gender are shown in Table 2.

According to the findings presented in Table 2, there were no significant differences between boys and girls in terms of their use of estimation strategies across the weight, length, and area measurement problems. In the weight measurement problem, both boys and girls showed an equal inclination towards using the segmentation/subdividing/chunking strategies. Similarly, in the length measurement problem, both genders exhibited a similar preference for the unit iteration/separation strategy. Additionally, in the area measurement problem, boys and girls equally favored the segmentation/subdividing/chunking strategies. These results suggest that gender did not play a significant role in determining the measurement estimation strategies used by the students ($Weight\ Measurement_{Pearson\ \ Chi\text{-}Square} = 0.912$; $Length\ Measurement_{Pearson\ \ Chi\text{-}Square} = 0.300$; $Area\ Measurement_{Pearson\ \ Chi\text{-}Square} = 0.423$). Furthermore, the students were asked if they had any estimation strategies to suggest for the given

scenarios other than the concept cartoon scenarios. The strategies proposed by the participating students are presented in Table 3.

Upon reviewing Table 3, it is evident that students predominantly relied on their previous experiences or prior knowledge when generating estimation strategies for the weight measurement estimation problem. Additionally, it was observed that students often opted for alternative solution-oriented approaches instead of producing direct estimations. Regarding the estimation strategies generated for the length measurement estimation problem, students showed a preference for the unit iteration/separation strategy and measuring with standard units over other options. When examining the strategies produced for the area measurement estimation problem, it is apparent that responses involving measurements in standard units were more frequently provided compared to other strategies.

Table 2 The measurement strategies used by the students according to their gender.

Estimation strategies	Weight measurement estimation		Length measurement estimation		Area measurement estimation	
	Boy	Girl	Boy	Girl	Boy	Girl
Reference point/ comparison	6	6	4	5	2	6
Segmentation/ subdividing/ chunking	13	13	6	5	8	8
Prior knowledge	2	3	5	1	8	5
Unit iteration/ separation	2	1	8	12	5	4

These findings highlight the importance of students' prior experiences and knowledge in their estimation strategies. Students tend to draw on their existing understanding and familiarity with weight, length, and area concepts when making estimations. This suggests that incorporating real-life examples and practical applications in mathematics education can support students in developing more accurate and meaningful estimation strategies. The prominence of alternative solution-oriented approaches in weight estimation indicates that students may employ problem-solving techniques beyond direct estimations. This highlights the need to encourage flexible thinking and diverse problem-solving strategies in mathematics instruction. The prevalence of unit iteration/ separation and measuring with standard units in length estimation reflects students' recognition of the significance of using known units and breaking down the measurement into smaller parts. It underscores the role of standard units as reference points for estimation and the value of utilizing measurement tools effectively. The emphasis on measurement in standard units in the area estimation problem suggests that students recognize the significance of using standardized measurements for accurate estimations. This underscores the importance of developing students' understanding of measurement units and their application in different contexts. Overall, these findings contribute to our understanding of students' estimation strategies in measurement tasks and emphasize the need for targeted instruction that builds on students' prior knowledge, promotes problem-solving skills, and enhances their understanding of measurement concepts and units.

Discussion and conclusion

In the present study, the primary objective was to investigate the measurement estimation strategies of fourth-grade students in a classroom setting using concept cartoons. A total of 46 students

Table 3 The estimation strategies produced by the students and the students' answers.

Scenario (categories)	Estimation strategies (codes)	*f*	Students' answers
Weight Measurement Estimation	Prior knowledge	9	S1: I estimate by looking at their length and height myself
	Another solution-oriented option	5	S5: He can ask another grocery store and measure the watermelon with his scale.
	Reference point/comparison	3	S19: I hold it in my hand and compare the weights of the two
	Irrelevant answer	2	S16: We used to go to the market with my family and measure
	Unit iteration/separation	1	S24: I would count 10 grams per line of watermelon.
	Total	20	
Length Measurement Estimation	Unit iteration/separation	10	S22: First I measure with my own fathom, I see how many meters a fathom is, and then I measure by putting my fathom on top of each other.
	Measuring in standard units	6	S11: I will bring a measuring tape and measure it
	Segmentation/subdividing/ chunking	1	S21: I used to divide half of the cabinet with my hand and calculate the size of the cabinet like that
	Reference point/comparison	1	S15: I would find a brother as tall as the door. Then I would measure that brother.
	Another solution-oriented option	1	S9: I would ask the other class teacher for the door size
	Irrelevant answer	2	S10: I can measure and calculate the top and bottom parts first and then the sides.
	Total	21	
Area measurement estimation	Measuring in standard units	3	S36: I used to take a long ruler and measure its length.
	Irrelevant answer	3	S31: I say to our family member or an elderly person: can we increase the length of one click?
	Unit iteration/separation	2	S24: I think it can be measured by fathom or steps.
	Reference point/comparison	2	S40: I think it is as much as the garden of our apartment.
	Segmentation/subdividing/ chunking	2	S23: Can measure and find seats and spaces
	Prior knowledge	1	S7: They can find it by looking at the movie theater.
	Another solution-oriented option	1	S16: We can find the answer by asking the owner of the movie theater.
	Total	14	

participated and were presented with realistic problem situations related to weight, length, and area measurements. The students were asked to identify which answer they believed was closest to the correct estimation based on their own understanding and experiences. Furthermore, it was aimed to explore how students would respond if they were faced with the same measurement problem. The idea was to understand students' personal approaches and thought processes when tackling measurement estimations. By engaging students in this manner, it was aimed to identify their individual perspectives and gather insights into their decision-making processes regarding measurement estimations. This approach provided valuable information on students' reasoning abilities and problem-solving strategies and their ability to apply measurement concepts to real-life situations.

The aim of using concept cartoons in the present study was to provide students with a means to externalize their mathematical thinking processes, allowing them to express their thoughts freely and establish connections with other ideas in a familiar environment. By presenting realistic problems through concept cartoons, students were immersed in scenario-based situations that motivated them to find solutions. This approach aligns with the principles of RME and concept cartoons, as it promotes active student engagement and the application of mathematical concepts in real-life contexts. The analysis of the measurement estimation strategies provided by the students revealed that, in general, the most favored strategies in all scenarios were "Segmentation/sub-segmentation/fragmentation" and "Unit iteration/separation". In previous studies, it was also shown that students used the "Unit iteration/separation" strategy at rates ranging from 30% to 97% (Hildreth, 1983; Immers, 1983). Additionally, Lehrer et al. (2003) emphasized the importance of students using the "Reference point/comparison" strategy before employing standard measuring instruments, as it allows them to model the process of unit iteration physically and then mentally. However, it was observed that fourth-grade students showed less interest in this strategy, which contrasts with previous findings indicating its significance. It can be inferred that comparison or reference points are important for estimation, but students tend to use this strategy less frequently (Hildreth, 1983). In a pilot study conducted by Joram et al. (2005) with third-year students, it was also found that the "Reference point/comparison" strategy was rarely utilized for measurement estimation. These findings suggest that there may be variations in the strategies employed by students at different grade levels, highlighting the need for further investigation into students' understanding and utilization of estimation strategies in the context of measurement. Such variations in findings could be attributed to the mathematics teaching programs and textbooks that each country employs. The extent to which these programs and textbooks emphasize estimation strategies will influence the diversity of strategies students use. Analyzing the results of the research conducted by Bulut and colleagues (2017) on the examination of estimation skills in mathematics teaching programs between 1945 and 2015, it is noteworthy that there are a 'limited number of studies on estimation' and 'insufficiency in the inclusion of estimation skills in the curriculum in Turkey.' As of 2018, the Ministry of National Education has started to emphasize estimation more in the mathematics teaching program (MoNE, 2018). Consequently, the number of studies conducted on estimation in the country between 2017 and 2019 has increased compared to other years (Bağdat and Yıldız, 2023). Although the concept of estimation is present in the programs, it has been limited to certain grade levels. In particular, due to the necessity for students to have sufficient prior knowledge about the topic, it has been somewhat neglected at the K-4 level. Moreover, research indicates that in textbooks the only estimation strategy mentioned is usually rounding. Additionally, textbooks lack discussions about various estimation strategies and there is a lack of strategy instruction for students.

Another notable finding in the study was the similarity in measurement estimation strategies used by male and female students. While there is limited research specifically exploring gender differences in measurement strategies, Hildreth (1983) conducted a study involving students at different grade levels (fifth, seventh, and university first grade) and found no significant differences in the measurement strategies employed by students based on their gender. These results suggest that gender may not play a significant role in influencing the choice of measurement estimation strategies among students. However, further research is needed to delve deeper into this aspect and explore potential factors that might contribute to variations in strategy preferences.

In a study conducted by Ruwisch et al. (2015) involving fourth-grade students, it was discovered that these students employ a variety of strategies to estimate lengths and areas. These strategies were categorized into three main types: comparisons, division into units, and segmentation/grouping. Interestingly, the students demonstrated equal use of these strategies for both length and area measurement estimations. The students also exhibited spontaneous generation of measurement estimation strategies, including prior knowledge, another solution-oriented approach, reference point/comparison, irrelevant answers, unit iteration/separation, measuring in standard units, and segmentation/subdividing/chunking. This diversity in strategies indicates flexibility in their mathematical thinking, showcasing their ability to adapt different approaches based on the context of the measurement problem (Drijvers et al., 2019). When examining the strategies generated by the students in the present study, it was observed that their approaches to measuring length and area were remarkably similar. However, considering the meaningfulness of mathematical ideas when connected to other concepts and situations (Clements and McMillen, 1996), it was hypothesized that students would be influenced by concept cartoons and potentially develop different strategies in their original responses. Moreover, the observation of students demonstrating equal use of strategies for both length and area measurements implies a certain level of transferability in their problem-solving skills across different mathematical domains. This not only shows the versatility of students' cognitive processes but also suggests that interventions targeting measurement estimation skills may have broad applications (Desli and Giakoumi, 2017). The spontaneous generation of measurement estimation strategies, including prior knowledge, another solution-oriented approach, reference point/comparison, irrelevant answers, unit iteration/separation, measuring in standard units, and segmentation/subdividing/chunking, underscores the creative and exploratory nature of students' problem-solving processes. It indicates that students draw upon various cognitive resources, including their existing knowledge, to formulate strategies when faced with measurement estimation challenges (Tambychik and Meerah, 2010).

Notably, the presence of a substantial number of "other solution-oriented options" and "irrelevant answers" in the students' responses should not be overlooked. These responses highlight the need for cultivating accuracy and relevance in their estimation strategies. As the strategies employed by children play a vital role in problem-solving, they must possess accurate and effective estimation strategies (Peeters et al., 2016). Hildreth (1983) emphasizes the importance of incorporating student discussions during training sessions focused on the utilization of measurement estimation strategies. Classroom examples can be utilized to illustrate each strategy and provide students with a deeper understanding. It is important to note that one limitation of the present study is the use of concept cartoons for evaluation purposes rather than instructional purposes. Consequently, the study lacked the inclusion of desired group activities, discussions, and additional examples based on similar scenarios and new

problems. During the study, students were instructed to select the strategy that closely aligned with their own preferences by considering the measurement strategies presented in the speech bubbles of the concept cartoons. In this approach, the aim was to enhance their awareness of their own measurement estimation strategies and potentially inspire the development of new and spontaneous approaches, perhaps influenced by the strategies depicted in the concept cartoons.

In conclusion, in the present study, the measurement estimation strategies employed by fourth-grade students using concept cartoons and realistic problem scenarios were investigated. It was revealed that students utilize various strategies, including comparisons, division into units, and segmentation/grouping, to estimate lengths and areas. The findings emphasize the significance of connecting mathematical concepts with real-life situations. Additionally, research shows that incorporating real-life contexts into mathematical problems can significantly improve student understanding and application of concepts (Adams and Lowery, 2007). The aim of using concept cartoons was to influence students' original answers and encourage the adoption of different strategies. However, the presence of a notable number of "other solution-oriented options" and "irrelevant answers" in the students' responses highlights the need for accurate and useful estimation strategies. This suggests an area for instructional focus, emphasizing the importance of guiding students toward strategies that not only showcase creativity but also align with the requirements of the given mathematical problem (Heinze et al., 2009). The research underscores the importance of incorporating student discussions and examples in teaching measurement estimation strategies. By integrating these elements into instructional sessions, students can enhance their understanding and application of various strategies.

Limitations and suggestions. While the study contributes valuable insights into students' measurement estimation strategies, it has certain limitations. Notably, the focus on evaluation rather than teaching limited the inclusion of group activities, discussions, and additional examples. Addressing these limitations and further refining the study can strengthen its contribution to the field of mathematics education. Overall, the study provides a comprehensive analysis of students' use of measurement estimation strategies in the context of concept cartoons and realistic problems. The findings underscore the importance of promoting accurate and effective estimation strategies among students, highlighting the potential for further research and instructional improvements in this area.

As suggestions for researchers and future studies, educators, and their practices, several recommendations can be made based on the findings and limitations of the present study:

Suggestions for researchers and future studies:. Several recommendations can be proposed for researchers and future studies based on the findings and limitations of the present study. Expanding the research scope by conducting similar studies with larger sample sizes and across different grade levels can provide a more comprehensive understanding of students' measurement estimation strategies, allowing comparisons and analysis of developmental changes in strategy choices. Longitudinal studies tracking students' progression in measurement estimation strategies can shed light on the development and effectiveness of different strategies over time. Qualitative analysis, such as interviews or think-aloud protocols, can supplement quantitative data, providing deeper insights into students' thought processes, reasoning, and decision-making strategies during measurement estimation tasks. Intervention studies that explicitly target measurement estimation strategies can assess their effectiveness in promoting accurate and useful estimation approaches among students. Comparing the outcomes of different instructional methods or interventions can provide evidence-based guidance for educators. Conducting studies that focus on integrating concept cartoons and realistic problem scenarios into classroom instruction rather than solely using them for evaluation purposes can offer valuable insights into the impact of such interventions on students' strategy preferences and problem-solving abilities. Investigating potential differences in measurement estimation strategies based on various demographic factors, such as gender, cultural background, or mathematical achievement, can inform personalized instruction and support equitable mathematics education.

Implications for educators. The present study offers valuable insights that educators can use to enhance their instructional practices in the area of measurement estimation strategies. The findings underscore the significance of incorporating realistic problem scenarios, such as those presented through concept cartoons, into classroom instruction. Educators can adopt a pedagogical approach that aligns with the principles of Realistic Mathematics Education (RME) and concept cartoons, promoting active student engagement and the application of mathematical concepts in real-life contexts. By immersing students in scenario-based situations, educators can stimulate their problem-solving abilities and foster a deeper understanding of measurement estimation strategies. Moreover, the study emphasizes the importance of recognizing and addressing the diversity of strategies employed by students. The principles of RME and concept cartoons can guide educators in providing differentiated instruction that accommodates varied learning styles and preferences. The study also highlights the need for explicit instruction and discussions on measurement estimation strategies. Practically, educators can integrate the use of concept cartoons into lesson plans to facilitate discussions and activities focused on measurement estimation. Educators can incorporate classroom activities that encourage students to articulate their thought processes, share their strategies, and engage in collaborative problem-solving. By creating an environment that values diverse strategies, educators can contribute to the development of students' reasoning abilities and problem-solving skills. Classroom activities can be designed to encourage group discussions, peer interactions, and additional examples related to the concept cartoons presented. In essence, this study opens up avenues for educators to refine their teaching methods, promote a variety of measurement estimation strategies, and create inclusive learning environments that nurture students' mathematical reasoning.

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author upon reasonable request.

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Author contributions

The author confirms sole responsibility for the following: study conception and design, data collection, analysis and interpretation of results, and manuscript preparation.

Competing interests

The author declares no competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Ethical approval

The questionnaire and methodology for this study were approved by the Ethical Committee of Çukurova University (Ethics approval number: 21411074).

Informed consent

In this study, freely given informed consent to participate in the research was obtained from the participants and their parents and will be presented in manuscript form upon request.

Additional information

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